NOISE SUSCEPTIBILITY OF PHASE UNWRAPPING ALGORITHMS FOR INTERFEROMETRIC SYNTHETIC APERTURE SONAR

S. Banks, T. Sutton, H. Griffiths

Department of Electronic & Electrical Engineering, University College London, Torrington Place, London WC1E 7JE.
email: s.m.banks@ucl.ac.uk

Interferometric Synthetic Aperture Sonar (ISAS) offers the possibility of high resolution, three-dimensional images of objects on the ocean floor. The height of objects is identified from the phase difference between two synthetic aperture images of the seabed obtained using two arrays separated by a short distance. A central problem is the need to unwrap the phase difference in the presence of noise to obtain an unambiguous height estimate.

This paper presents an end-to-end simulation of the imaging of a three-dimensional object on the seabed. The susceptibility of this image to reverberation-type noise is evaluated, and the effectiveness of different forms of phase unwrapping algorithms are compared.

1. INTRODUCTION

Interferometry provides a means of reconstructing a three-dimensional representation of a target scene using sonar. Interferometry has been successfully applied to both satellite and aircraft mounted radar; these techniques are now being adapted to Interferometric Synthetic Aperture Sonar (ISAS).

ISAS is inherently more challenging than ISAR due to increased platform movement, slower and variable propagation of waves in the medium and much increased reverberation. This work is part of a collaboration between UCL, Thompson Marconi Sonar, DERA and EPSRC to develop improved algorithms for motion compensation, imaging and identifying targets on the seabed. A detailed simulation of the whole sonar process, starting from the expected received reflections off a target scene is described. Image reconstruction techniques are developed using this simulated data.
2. OVERVIEW OF THE INTERFEROMETRIC SYNTHETIC APERTURE SONAR PROCESS

1.1. Synthetic aperture systems.

With a real aperture sonar system, a long array of hydrophones gives a higher resolution than a short array. Synthetic aperture sonar systems employ one short array of hydrophones, and combine the responses from that array at several different locations to create a very long synthetic aperture. The azimuthal resolution of such a system can be shown to be independent of range, and equal to half the length of the real aperture [5].

1.2. Height estimation from interferometry

An estimation of the relative height of a target scene may be obtained using interferometry. A height estimation is derived from the phase difference between two images of the target scene taken from different receivers. The geometry is shown in Figure 1.

The process is complicated by phase ambiguity – when the phase difference between the two received signals is greater than \(2\pi\) radians. The problem of resolving phase ambiguity is exacerbated by noise, platform movement, layover and shadowing. Phase ambiguity may be removed by phase unwrapping, discussed later.

The relative height of the target scene can be calculated from the unwrapped phase difference between the upper and lower image.

If \(\phi\) is the phase difference between the received signal at RX1 and the received signal at RX2 then from Figure 1:

\[
r_z = r_i + \frac{\phi \lambda}{2\pi}
\]  

where \(\lambda\) is the wavelength of the emitted chirp. From the geometry:

\[
\theta = \cos^{-1}\left(\frac{r_z^2 - B^2 - r_i^2}{2 * B * r_i}\right) - \alpha
\]

\[
h = H - r_i \cos \theta
\]
3. SIMULATING THE RESPONSE FROM A TARGET SCENE

Software has been written to simulate the sonar reflections off a given target scene. This provides a platform to evaluate phase unwrapping algorithms and the effects of noise. The simulation represents the scene using a finite number of point reflectors spaced at approximately the distance of one wavelength of the emitted chirp. Closer spacing results in a smooth reflective surface, which is hard to detect using sonar as most of the reflected signal is reflected away from the receiver. This will be the subject of future work.

The simulation process is computationally intensive, as each point reflector must be simulated for each element of both arrays for each ping. For 2048 sample points, 250*120 point reflectors, 32 channels and 64 pings the simulation takes around 2 hours using two Sun Ultra 10s (one for each receiver).

4. THE IMAGE CREATION PROCESS

Creating an image of the target scene from the simulated data consists of several distinct steps. This section describes these steps.

1.3. Formation of two two-dimensional images

Images corresponding to the received signal from each receiver are formed using synthetic aperture processing. Several techniques exist; for the simulation the chirp-scaling algorithm is used [4].

1.4. Co-registration of two images

The two images formed from the upper and lower receivers are with respect to \( r_1 \) and \( r_2 \) respectively and do not lie on top of each other. They must be co-registered to within one pixel to enable successful height estimation. Several techniques, such as cross-correlation [6] exist that require no a-priori knowledge of the target scene exist; however such techniques have not been used. For the purposes of the simulation, the two images are co-registered from the known geometry of the scene. The scene is assumed flat.

The geometry of the system is shown in Figure 1. For a given pixel, the distances \( r_1 \) and \( r_2 \) to a point A may be calculated from the respective reconstructed images. Therefore, the image formed from RX2 may be co-registered with that formed from RX1 using the expression (4) for \( (r_2-r_1) \) in terms of \( r_1 \).

\[
(r_2 - r_1) = \sqrt{R + \left(\sqrt{r_1^2 - H^2} - S\right)^2} - r_1
\]

where: \( S = drx \sin \alpha \) and \( R = (H + drx \cos \alpha)^2 \)

The image from the upper receiver can be co-registered with that from the lower receiver by moving each pixel in the upper image the equivalent distance \( (r_2-r_1) \). To ensure the co-
registration works to within one pixel, the resolution of the upper image must be increased by up-sampling [2]. Note the images produced are complex images; hence, to retain phase information both the real and imaginary parts must be co-registered.

1.5. Obtaining an interferogram

An interferogram of the target scene is obtained by subtracting the phases of the two correlated images. Figure 2 shows the two images obtained for a simulated Gaussian mound, along with the resulting interferogram. The colour of the image represents phase, the brightness amplitude.

![Figure 2](image)

*Figure 2 (a) Phase of image produced from upper receiver, (b) phase of image produced by lower receiver, (c) interferogram i.e. the phase difference between (a) and (b)*

1.6. Phase unwrapping and phase correction

The phase value shown on the interferogram is modulo $2\pi$. However, the real phase difference between the two images may be several multiples of $\pi$. Assuming the interferogram is error free, the real phase can be calculated using phase unwrapping. Several phase unwrapping techniques exist; the technique described below was first proposed in [3].

For a one-dimensional array, phase unwrapping is straightforward. Take for example the following normalised phase values:

\[
\begin{array}{cccccccc}
0.4 & 0.7 & 0.0 & 0.1 & 0.8 & 0.5 & 0.2 \\
\end{array}
\]

The phase in the interferogram is assumed to vary no more than 0.5 of a cycle between adjacent pixels. Therefore, a corresponding maximum gradient of the target scene for successful imaging exists. It is clear that to unwrap the phase, going from left to right, 1.0 must be added to the third and fourth values in the above example. The same result is obtained by going from right to left.

![Figure 3](image)

*Figure 3 Positive and negative residues*
For a two-dimensional image, the phase can be unwrapped in a similar manner so long as discontinuities known as residues are not present [3]. A residue occurs when the integration around a 4-pixel square has a value other than zero, as shown in Figure 3. The residues are marked as either positive or negative as appropriate.

Residues of a given sign are linked to nearby residues of the opposite sign, or the edge of the image whichever is closer. The line between the residues is known as a cut. The algorithm described in [3] is used in the simulation to link the residues. This algorithm is summarised below.

A residue is searched for. When a residue is found, a box is extended around the residue until a new residue of opposite sign is found. When such a residue is found, the two are linked with a cut line and the search continues for another residue. If the edge of the image is found before a residue of the opposite sign, the residue is linked to the edge of the image. The assumption that the nearest residue of opposite sign is off the edge of the image is made.

The phase of the image may be successfully unwrapped using any path so long as the unwrapping path does not cross a cut line.

The detection of, and avoidance of phase-unwrapping errors caused by residues is discussed in several papers, including [1] and [3]. Residues may occur if the image is noisy or the target scene contains geometries that cause shadowing or layover.

For the simulation, residues have been generated by adding noise to a part of the interferogram. Figure 4 shows the interferogram with 3.2 peak phase noise added to a selected part, the resulting residues and the resulting cuts made to link the residues. The cuts were made using the algorithm described in [3].

![Figure 4](image1.png)

*Figure 4* (a) Noisy part of interferogram, (b) residues: white = -1, black = +1, (c) linked residues

1.7. Calculating scene height from phase-unwrapped interferogram

The relative height of the target scene is calculated from the phase-unwrapped interferogram using the method described in 1.1. Figure 5a shows the height derived from the unwrapped phase of the interferogram shown in Figure 2c. The phase has been unwrapped in straight lines starting from each side of the image. Phase unwrapping stops when a cut line is reached. This algorithm, although simple, works well when the number of cut lines is low.

Figure 5b shows the same image phase unwrapped in the right to left direction only, ignoring the cut lines. The diagram shows how errors resulting from unwrapping past a cut line are propagated across the image.
Figure 5(a) Reconstructed 3D image with noise  (b) Errors propagated globally when cut lines are ignored

REFERENCE


